Efficiency optimization in a correlation ratchet with asymmetric unbiased fluctuations

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(Received 8 September 2003; published 24 December 2003)

The efficiency of a Brownian particle moving in a periodic potential in the presence of asymmetric unbiased fluctuations is investigated. We found that even on the quasistatic limit there is a regime where the efficiency can be a peaked function of temperature, which proves that thermal fluctuations facilitate the efficiency of energy transformation, contradicting the earlier findings [H. Kamegawa *et al.*, Phys. Rev. Lett. **80**, 5251 (1998)]. It is also found that the mutual interplay between temporal asymmetry and spatial asymmetry may induce optimized efficiency at finite temperatures. The ratchet is not most efficient when it gives maximum current.

DOI: 10.1103/PhysRevE.68.061105 PACS number(s): 05.40.−a, 02.50.Ey, 87.10.+e

I. INTRODUCTION

For some years the problem of noise-induced transport has attracted much interest in theoretical as well as experimental physics [1–4]. This subject was motivated by the challenge to explain unidirectional transport in biological systems, as well as their potential technological applications ranging from classical nonequilibrium models [4,5] to quantum systems [6–8]. Several models have been proposed to describe a muscle's contraction [9–11], or the asymmetric polymerization for actin filaments responsible for cell motility [1,12].

The rectification of noise leading to unidirectional motion in ratchet systems has been an active field of research over the last decade. In these systems the directed Brownian motion of particles is induced by nonequilibrium noise in the absence of any net macroscopic forces and potential gradients. Several physical models have been proposed: rocking ratchets [13], fashing ratchets [14], diffusion ratchets [15], correlation ratchets [16], etc. In all these models the potential is taken to be asymmetric in space. For these nonequilibrium systems external random force should be time asymmetric or the presence of space dependent mobility is required.

In recent years the energetics of these systems, which rectify the zero-mean fluctuations, are investigated [17–19]. To define optimal models for such ratchet systems, the maximization of the efficiency of energy transformation is necessary. Much of the interest was motivated by the elegant piece of work done by Magnasco [13], which showed that a Brownian particle, subject to external fluctuations, can undergo a nonzero drift while moving under the influence of an asymmetric potential. The temperature dependence of the current has been studied and it has been shown that the current can be a peaked function of temperature. He claimed that there is a region where the efficiency can be optimized at finite temperatures and the existence of thermal fluctuations facilitate the efficiency of energy transformation. His claim is interesting because thermal noise is usually known to disturb the operation of machines. Based on energetic analysis of the same model Kamegawa *et al.* [19] made an important conclusion—that the efficiency of energy transformation cannot be optimized at finite temperatures and that the thermal fluctuations do not facilitate it. However, the discussion in that paper was only on the quasistatic limit, where the change of the external of force is slow enough and Takagi and Hondou [20] found that thermal noise may facilitate the energy conversion in the forced thermal ratchet when the ratchet is not quasistatic. Recently, investigation of Dan *et al.* [17] showed that the efficiency can be optimized at finite temperatures in inhomogeneous systems with a spatially varying friction coefficient in an adiabatically rocked rachet, and efficiency optimization in homogeneous nonadiabatical

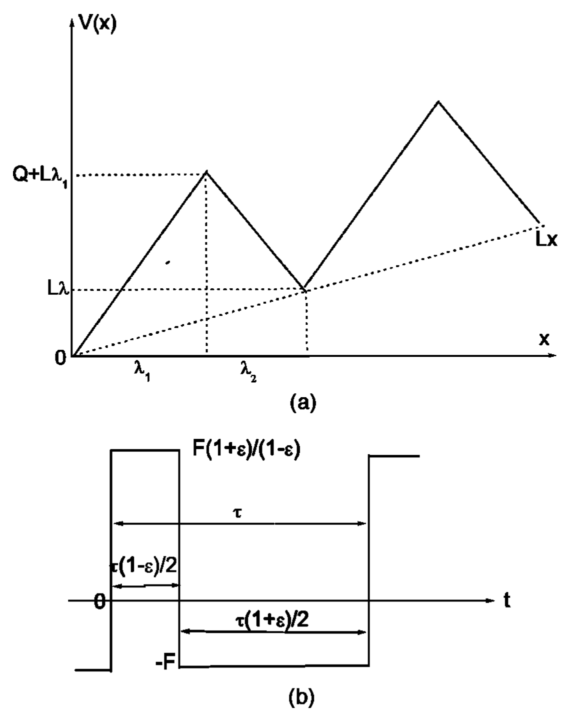

FIG. 1. (a) Schematic illustration of the potential, $V(x) = V_0(x)+V_L(x)$; $V_0(x)$ is a piecewise linear and periodic potential. V_L is a potential due to the load. The period of the potential is $\lambda=\lambda_1+\lambda_2$, and $\Delta=\lambda_1-\lambda_2$. (b) The driving force $F(t)$ which preserved the zero mean $\langle F(t)\rangle=0$, where the temporal asymmetry is given by the parameter ε.

rachet systems was observed by Sumithra *et al.* [18]. The question of whether the thermal fluctuations actually facilitate the energy transformation in forced homogeneous adiabatical ratchet systems is still unknown and it is the subject of the current investigation.

II. THE MODEL

We consider a rocking ratchet system subject to an external load,

$$\frac{dx}{dt}=-\frac{\partial V_0(x)}{\partial x}-\frac{\partial V_L(x)}{\partial x}+F(t)+\sqrt{2k_BT}\xi(t), \tag{1}$$

where x represents the position of the ratchet. V_0 is a periodic potential, $\xi(t)$ is a randomly fluctuating Gaussian white noise with zero mean and with the autocorrelation function $\langle\xi(t)\xi(s)\rangle=\delta(t-s)$. Here $\langle\cdots\rangle$ denotes an ensemble average over the distribution of the fluctuating forces $\xi(t)$. V_L is a potential against which the work is done and $\partial V_L/\partial x=L>0$ is the load force. The geometry of the potential $V(x)=V_0(x)+V_L(x)$ is displayed in Fig. 1(a). $F(t)$ is some external driving force which is shown in Fig. 1(b). The evolution of the probability density for x is given by the associated Fokker-Planck equation,

$$\frac{\partial P(x,t)}{\partial t}=\frac{\partial}{\partial x}\left[k_BT\frac{\partial P(x,t)}{\partial x}+[V'(x)-F(t)]P(x,t)\right]$$
$$=-\frac{\partial j}{\partial x}. \tag{2}$$

If $F(t)$ changes very slowly, there exists a quasistationary state. In this case, the average current of the particle can be solved by evaluating the constants of integration under the normalization condition and the periodicity condition of $P(x)$, and the current can be obtained and expressed as [13]

$$j(F(t))=\frac{P_2^2\sinh\{\lambda[F(t)-L]/2k_BT\}}{k_BT(\lambda/Q)^2P_3-(\lambda/Q)P_1P_2\sinh\{\lambda[F(t)-L]/2k_BT\}} \tag{3}$$

where

$$P_1=\Delta+\frac{\lambda^2-\Delta^2}{4}\frac{F(t)-L}{Q}, \tag{4}$$

$$P_2=\left(1-\frac{\Delta[F(t)-L]}{2Q}\right)^2-\left(\frac{\lambda[F(t)-L]}{2Q}\right)^2, \tag{5}$$

$$P_3=\cosh(\{Q-0.5\Delta[F(t)-L]\}/k_BT)$$
$$-\cosh\{\lambda[F(t)-L]/2k_BT\}, \tag{6}$$

where $\lambda=\lambda_1+\lambda_2$ and $\Delta=\lambda_1-\lambda_2$. The average current, the quantity of primary interest, is given by

$$J=\frac{1}{\tau}\int_0^\tau j(F(t))dt, \tag{7}$$

where τ is the period of the driving force $F(t)$, which is assumed longer than any other time scale of the system in this adiabatic limit. Magnasco considered this case, but only for $F(t)$ symmetric in time. Here we will again consider a driving with a zero mean, $\langle F(t)\rangle=0$, but which is asymmetric in time [21]

$$F(t)=\frac{1+\varepsilon}{1-\varepsilon}F[n\tau\leqslant t<n\tau+\tfrac{1}{2}\tau(1-\varepsilon)], \tag{8}$$

$$=-F[n\tau+\tfrac{1}{2}\tau(1-\varepsilon)<t\leqslant(n+1)\tau]. \tag{9}$$

In this case the time-averaged current is easily calculated,

$$J=\tfrac{1}{2}(j_1+j_2), \tag{10}$$

where $j_1=(1-\varepsilon)j\{[(1+\varepsilon)/(1-\varepsilon)]F\}$ and $j_2=(1+\varepsilon)j(-F)$.

The input energy R per unit time from an external force to the ratchet and the work W per unit time that the ratchet system extracts from the fluctuation into the work are given, respectively [19],

$$R=\frac{1}{t_j-t_i}\int_{x(t_i)}^{x(t_j)}F(t)dx(t), \tag{11}$$

$$W=\frac{1}{t_j-t_i}\int_{x(t_i)}^{x(t_j)}dV(x(t)). \tag{12}$$

For the square wave, they yield

$$\langle R\rangle=\tfrac{1}{2}F(j_1-j_2), \tag{13}$$

$$\langle W\rangle=\tfrac{1}{2}L(j_1+j_2). \tag{14}$$

Thus the efficiency η of the system to transform the external fluctuation to useful work is given by

$$\eta=\frac{\langle W\rangle}{\langle R\rangle}=\frac{L(j_1+j_2)}{F(j_1-j_2)}, \tag{15}$$

which in turn, being $j_2/j_1<0$, can be written as

$$\eta=\frac{L}{F}\left(\frac{1-|j_2/j_1|}{1+|j_2/j_1|}\right). \tag{16}$$

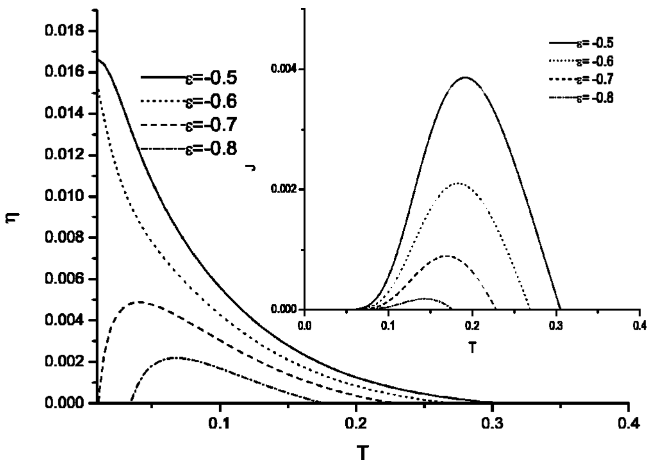

FIG. 2. Efficiency η as a function of temperature T for different values of asymmetric parameters $\varepsilon=-0.5,-0.6,-0.7,-0.8$, $F=1.0$, $\lambda=1.0$, $\Delta=0.7$, $Q=1.0$, and $L=0.01$. The inset shows the net current J as function of T for the same parameters.

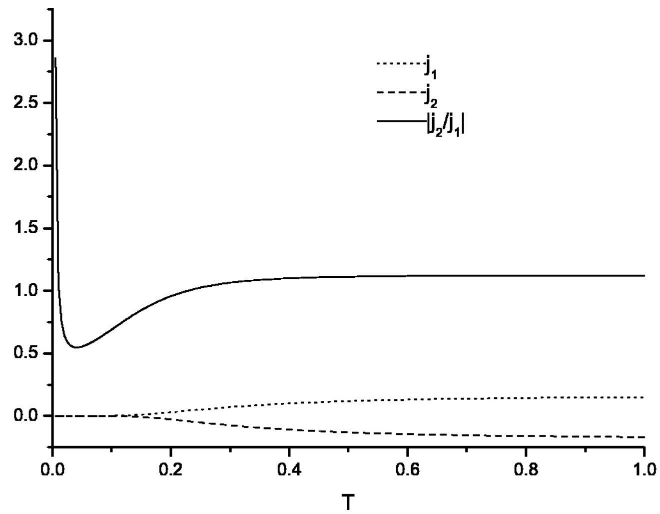

FIG. 4. Plot of currents j_1, j_2 and $|j_2/j_1|$. The condition is the same as the case $\varepsilon=-0.7$ in Fig. 2

III. RESULTS AND DISCUSSION

We have calculated the efficiency and the net current as a function of temperature T for the case where asymmetric unbiased fluctuations are applied, and the results are shown in Figs. 2–6.

In Fig. 2 we plot the efficiency η as a function of the temperature for different values of ε $(\varepsilon<0)$with the parameter values, $F=1.0$, $\lambda=1.0$, $\Delta=0.7$, $Q=1.0$, and $L=0.01$. From the figure we can see that the efficiency is a decreasing function of temperature for the cases of $\varepsilon=-0.5$ and $\varepsilon=-0.6$, which shows that the presence of thermal fluctuation does not help the efficient energy transformation by ratchet. But for the cases of $\varepsilon=-0.7$ and $\varepsilon=-0.8$ the efficiency can be optimized at finite temperatures. In contradiction with the results of Ref. [19] we found a region where the efficiency attains a maximum at a finite temperature. This shows that thermal fluctuations may facilitate the energy conversion for asymmetric unbiased fluctuations. The current is a peaked function of temperature for corresponding parameters as shown in the inset. The highest temperature of the ratchet decreases with the value of the asymmetric parameters ε of fluctuations and the lowest temperature of the ratchet does not change with the ε, which indicates that the asymmetric parameters are sensitive to the highest working temperature of the ratchet. The peak shifts to the lower temperature region with a decreasing value of the asymmetric parameters ε. Comparing Fig. 2 with the inset we can see that the temperature corresponding to the maximum current is not the same as the temperature at which the efficiency is maximum, which is consistent with the previous results [17–19].

From Eq. (16) we can know that the efficiency η depends on the ratio $|j_2/j_1|$. If the function is monotonically increasing, η should be a monotonically decreasing function of the temperature.

In Fig. 3 we plot the fluxes obtained for the case of $\varepsilon=-0.5$ (shown in Fig. 2). From Fig. 3 we can see that the

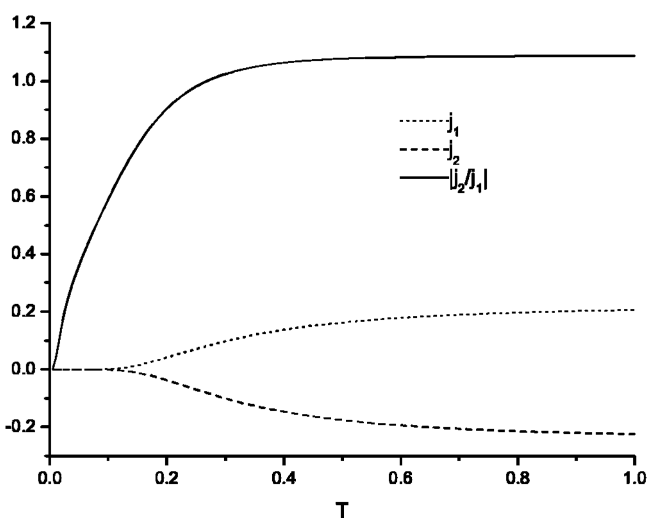

FIG. 3. Plot of currents j_1, j_2, and $|j_2/j_1|$. The condition is the same as the case $\varepsilon=-0.5$ in Fig. 2.

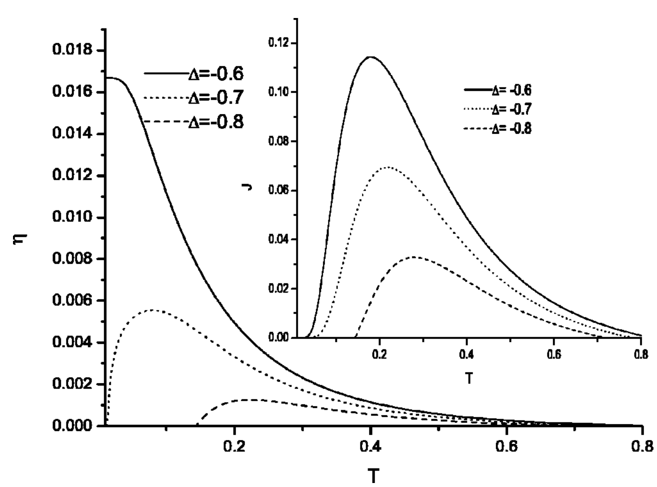

FIG. 5. Efficiency η as a function of temperature T for different values of asymmetric parameters $\Delta=-0.6,-0.7,-0.8$, $F=1.0$, $\lambda=1.0$, $\varepsilon=0.7$, $Q=1.0$, and $L=0.01$. The inset shows the net current J as a function of T for the same parameters.

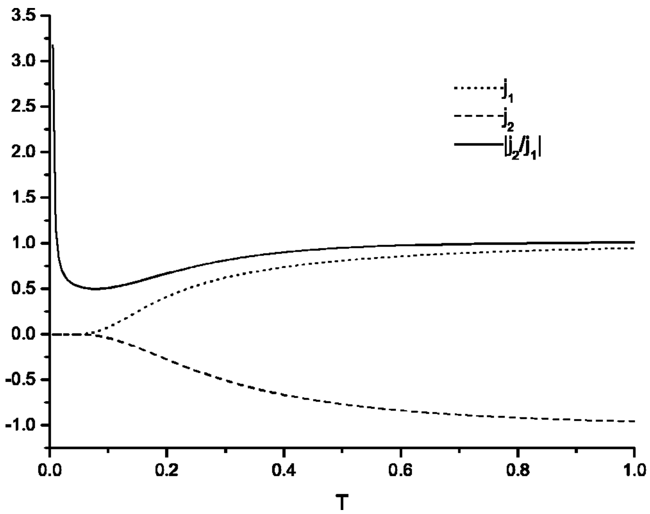

FIG. 6. Plot of currents j_1, j_2, and $|j_2/j_1|$. The condition is the same as the case $\Delta=-0.7$ in Fig. 5

ratio $|j_2/j_1|$ is a monotonically increasing function of temperature, which indicates that the efficiency η is a decreasing function of temperature. However, for the case of $\varepsilon=-0.7$ (see Fig. 4) the ratio $|j_2/j_1|$ displays a clear minimum at the same value of the temperature which corresponds to a maximum of η in Fig. 2.

In Fig. 5 we plot the efficiency η as a function of the temperature T for different values of the slope degree of potenial Δ ($\Delta<0$)with the parameter values $F=1.0$, $\lambda=1.0$, $\varepsilon=0.7$, $Q=1.0$, and $L=0.01$. From the figure, we can see that with the decreasing of Δ the efficiency function of the temperature becomes from a monotonically decreasing function to a peaked function. This shows that the thermal fluctuations actually facilitate the energy transformation in some region. The corresponding current is a peaked function of temperature for the same parameters as shown in the inset. The height of the peak decreases with the value of Δ. The lowest temperature of the ratchet changes with Δ drastically while the highest temperature of the ratchet does not change with the Δ and the peak shift to a higher temperature region with a decreasing value of Δ, which is opposite of the inset of the Fig. 2.

In Fig. 6, we plot the fluxes of the temperature for the case of $\Delta=-0.7$ (shown in Fig. 5). From Fig. 6 we can see that the ratio $|j_2/j_1|$ displays a minimum at the same value of the temperature, which corresponds to a maximum of η in Fig. 5.

IV. CONCLUSION

In the present paper the transport of a Brownian particle moving in a spatially asymmetric potential in the presence of asymmetric unbiased fluctuations is investigated. In contradiction with the previous findings, our results show that the mutual interplay between the asymmetry of fluctuation and the asymmetry of potential may cause an optimized efficiency of energy conversion. This proves the claim made by Magnasco that there is a regime where the efficiency can be optimized at finite temperatures. The temperature corresponding to the maximum current is not the same as the temperature at which the efficiency is maximum. The asymmetry ε of the fluctuation is sensitive to the high temperature working region of the ratchet while the asymmetry Δ of the potential affects the low temperature working region drastically.

In our paper the main features introduced by the temporal asymmetry are the interplay of lower potential barriers in a positive direction relative to the negative direction and in the corresponding shorter and longer times, respectively. These type of competitive effects appear ubiquitously in systems [21] where there is an interplay between thermal activation and dynamics.

ACKNOWLEDGMENTS

The project was supported by National Natural Science Foundation of China (Grant No. 10275099) and GuangDong Provincial Natural Science Foundation (Grant Nos. 021707 and 001182).

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